REVIEWS

Theorie schallnaher Strömungen. By K. G. GUDERLEY. Berlin: Springer-Verlag, 1957. 376 pp. DM. 42.

It is a real measure of the rapidity of technological progress that within the last fifteen years the transonic régime has passed from being a high-speed to a low-speed régime. The enormous body of literature on the subject is a tribute to the diligence of modern theoretical scientists. (It is an interesting question to ask if there is any connexion between the two phenomena.) In this book the author presents mainly the results of his efforts and those of his co-workers in transonic flow theory. Thus there is made available to the scientific public a substantial amount of detailed information which has appeared before only as U.S. Air Force Technical Reports. A reaction after a first glance at the book might be that a lot of mathematical heavy artillery is employed to shoot down a few pigeons, in the form of solutions to specific flow problems. Part of this impression is created by the author's desire to build up a complete mathematical background for the subject. However, the fact remains that even the simplest flow problems such as flow past an airfoil or body of revolution of arbitrary shape have resisted the barrage and furthermore there is no hint that solutions to these problems will be forthcoming by application of methods developed in the book. In spite of these comments, which are really an indication of the difficulty of solving the obvious problems for a non-linear partial differential equation, the reviewer feels that efforts along these lines are definitely worth-while. Every bit of progress in understanding details help to round out the general picture, build up the general confidence, suggest decisive experiments, etc. In the course of these investigations a surprising amount of technically useful information is obtained, such as similarity laws, asymptotic flow fields, and wind-tunnel wall effects. Also, the mathematical theory of the equations of mixed type which are studied is interesting in its own right. The advantage of having physical models to go with the equations is clearly brought out and explained by the author.

The main divisions of the book are indicated below, together with some comments:

I. *General basis.* A brief but clear introduction to inviscid compressible fluid theory.

II. *Simplification of the flow equations. Similarity law for transonic flow.* A brief treatment of the important concepts, which leads to an approximate transonic equation and the related similarity laws. The following approximate equation, and approximate boundary condition and shock relations, are obtained by a limit process in which, typically, thickness ratio $\tau \to 0$, free stream Mach number $M_\infty \to 1$ while $(1-M_\infty^2)/\tau^2$ is kept fixed, this procedure being carried out in a modified co-ordinate system where $x, \bar{y}\ (=\tau y), \bar{z}\ (=\tau z)$ are fixed:

$$(\gamma+1)\,\Phi_x\,\Phi_{xx} = \Phi_{\bar{y}\bar{y}} + \Phi_{\bar{z}\bar{z}} \quad (\Phi = \text{velocity potential}). \tag{1}$$

III. *Linearized treatment of transonic flow.* Brief discussion of the non-linear equation derived in II, showing that the solutions of the linearized version $\Phi_{\bar{y}\bar{y}}+\Phi_{\bar{z}\bar{z}} = 0$ cannot usually be made physically meaningful. The exception, the slender lifting wing of zero thickness, is also discussed.

IV. *Exact solutions of the potential equation of transonic flow.* Among these is a simple polynomial solution of (1), apparently new, which describes plane flow as it accelerates through sonic speed in the throat of a nozzle. This solution is used to introduce the very important concept, first noticed by Guderley, of the limiting Mach wave; this Mach wave bounds that part of the supersonic region which can influence the subsonic region. This idea, which occurred in hidden form in Tricomi's original purely mathematical treatment, here has a simple physical significance.

V. *Basis of the hodograph method.* A clear discussion is presented of the hodograph method for plane flow starting with simple incompressible examples. Tschapligin's solutions and their connexion with various asymptotic approximations, including the Tricomi approximation

$$\psi_{\eta\eta}-\eta\psi_{uu} = 0, \tag{2}$$

in which η and u are the hodograph variables, are discussed.

VI. *Discussion of transonic flow fields with the help of the hodograph.* Some simple solutions for flow over a corner and wedge in supersonic flow are presented, as well as some fairly complicated discussion of forked shock waves.

VII. *Particular solutions of the Tricomi equation.* The important solutions discovered by Guderley which have their singularity at $\eta = u = 0$ are presented in great detail. They are basically hypergeometric functions for which a special notation is introduced. The limiting Mach wave reappears as a co-ordinate and serves to separate physically meaningful solutions from the rest. The solutions of Tamada and Tomotika for singularities in the subsonic region and Falkowitsch for branch points are also discussed briefly. Certain complicated eigen-value problems are also discussed.

VIII. *Flow at Mach number* 1. This includes applications of the hodograph method to a number of problems first solved by Guderley. One of the Guderley solutions to the Tricomi equation is shown to correspond to the free stream and a special airfoil is produced by the addition of another particular solution. Too much detail is presented here. On the other hand, the solution for flow past a wedge is given a form which is brief compared with that given originally—more details could have been used here, especially with respect to the method of satisfying the boundary condition for flow around a corner. The solution for a wedge at angle of attack small compared to its thickness ratio is given, as well as that for a flat plate at angle of attack. An important practical result appears in the close agreement of the theoretical pressure distribution on the wedge at free stream Mach number 1 and that in a choked closed wind tunnel (derived later). No comment is made on whether or not the result might extend to shapes for which the sonic point can move with Mach number.

IX. *Flow fields at Mach numbers slightly different from* 1. A perturbation theory of flow at $M_\infty = 1$ is developed in detail. Choked wind tunnel problems are discussed.

X. *Further considerations of certain particular solutions, etc.*

XI. *Axi-symmetric flow.* Similarity methods are used to reduce (1) to a non-linear ordinary differential equation which describes the asymptotic flow field at a large distance from a body at $M_\infty = 1$. Group theory methods, similar to those used in blast wave theory, are used to obtain a first-order differential equation whose trajectories fix the flow. The asymptotic form is shown to be

$$\Phi = r^{-\frac{2}{7}} f\left(\frac{x}{r^{\frac{4}{7}}}\right),$$

but the scale of the solution cannot be related to any properties of the body. Unfortunately the discussion here is very brief, and so too is the following discussion of these asymptotic flows with shocks (Barish). A fairly complete list of references is presented.

As a criticism it might be said that the book does not present enough of the work of other authors. There are also some curious omissions of Guderley's own work, such as that on finite span wings. The pure mathematician would certainly find it not rigorous enough, while the engineer will no doubt find it too mathematical. There are no experimental points to mar the theoretical beauty (although transonic flow theory has produced many results in good agreement with experiment). Parts of the book are very difficult to read. Nevertheless, it is a good book representing work on a very high level and containing important practical and mathematical contributions. It is certainly a most important book for anyone who wants to try to solve transonic flow problems.

An English translation is to be made available shortly. J. D. COLE

Rheology: Theory and Applications, volume II. Edited by F. R. EIRICH. New York: Academic Press, 1958. 591 pp. $18.00.

Following a first volume which included some accounts of the theoretical analysis of various types of rheological behaviour, this second of three volumes on *Rheology: Theory and Application* lays greater emphasis on the phenomena actually observed in some real materials and on the methods available for their quantitative assessment. It is not intended to complete the picture of the present state of knowledge of deformation and flow behaviour, but is to be followed by a volume on other classes of materials of industrial importance and dealing with some technological developments in the rheological field.

The observation that in some real liquids the rate of shear is a constant multiple of the applied shear stress was noted by Newton, and this simple rule has been used by generations of physicists and applied mathematicians as a basis for their studies—experimental and theoretical—of fluid flow. Hooke's formulation of the observed proportionality of stress and strain has been similarly used as a foundation for the study of the mechanical properties of solids. Reading the volume under review, comprising thirteen chapters, each by a different author, one is reminded how physicists and physical chemists have been stimulated to act in rather different ways by the realization that some materials are neither Newtonian liquids nor Hookean solids. The physicist seeks to determine the precise mode of response to applied stress shown by a solid or liquid to which neither of the classical laws of Hooke and Newton can be applied. The physical chemist is more interesting in knowing how some simply-defined empirical parameter which can be associated with a material—usually a 'modulus' or a 'viscosity coefficient' which gives but an imperfect characterization of the system—varies with chemical constitution and environment. The contrast is between an intensive investigation carried out on a single specimen whose properties are not changing, and the repetition of a simple measurement on many specimens which are chemically related. Behind the second approach is the desire to obtain information about microscopic structure and molecular configurations (especially in macromolecular systems) from simple physical measurements; the chemist thinks of rheology largely as a means to an end, and rarely as an interesting study in itself.

The detailed assessment of a rheologically complicated material must await the development of suitable techniques to measure more than the simple 'modulus' or 'viscosity'—i.e. more than the ratio of a stress and a strain, or rate of strain, when the ratio is neither a constant nor a simple function. The progress of rheological investigations is therefore entirely dependent on the introduction of new techniques, the subject of the last three chapters in this volume: 'Experimental techniques for rheological measurements on viscoelastic bodies' by J. D. Ferry, 'Fundamental techniques—fluids' by B. A. Toms, and 'Goniometry flow and rupture' by A. Jobling and J. E. Roberts. There is surprisingly little overlapping of subject-matter in these three articles, and the different points of view on what one should measure in various types of material are welcome and stimulating.

Jobling and Roberts include an all-too-brief description of Roberts's method of measuring normal stresses in all directions (as well as the shear stress) during a steady-shearing experiment. It is to be regretted that more experimental details and more typical results of observation on some actual liquids have not been included. Several rival theories have grown up around the existence of normal stress-differences during shearing of elastico-viscous liquids, and real progress might be made in distinguishing them if more experimental results were made available.

In other chapters, experimental techniques peculiar to the systems considered are discussed incidentally. B. Gutenberg, in describing 'Rheological problems of the earth's interior' is clearly dependent on methods of observation peculiar to his field. This author brings it forcibly to the reader's notice that it is not only new man-made materials and natural polymers that give rise to mechanical response not explainable in terms of a single coefficient. At the same time, his contribution serves to remind the reader that if a single constant is insufficient, one might be able to manage with a small number of physical constants and get a reasonable theory—without necessarily jumping from one to a continuous infinity of parameters to characterize rheological behaviour, as is implicit in some other approaches.

Several chapters are concerned with polymer systems, and there is inevitably some overlapping, not always unwelcome, in the information presented. That the authors of different chapters do not use the same terminology or notation when writing about the same concepts is more disconcerting. The subject is introduced in a readable and informative opening chapter on 'Viscoelasticity phenomena in amorphous high polymeric systems' by H. Leaderman. Experimental results at small variable shear stresses applied to amorphous polymers can be explained in terms of a relaxation spectrum, or, for example, by functions of frequency measuring the in-phase and out-of-phase strain response to alternating stress, over the whole range of possible frequencies of application.

A closely related chapter on 'Stress relaxation studies of the viscoelastic properties of polymers' is by A. V. Tobolsky; and there follows one on 'The relaxation theory of transport phenomena' by T. Ree and H. Eyring, which might perhaps more fittingly have been found a place in volume I.

The remaining chapters in volume II each deal with a particular class of materials and with some special experimental techniques which have been developed in relation to those materials. They are 'The rheology of organic glasses' by R. Buchdahl, 'The rheology of raw elastomers' by M. Mooney, 'The rheology of cellulose derivatives' by E. B. Atkinson, 'The rheology of fibres' by R. Meredith, 'The rheology of gelatin' by A. G. Ward and P. R. Saunders, and 'Rheological Properties of Asphalts' by R. N. J. Saal and J. W. A. Labout. The emphasis on macromolecular systems, of one sort and another, is evident from the titles. In all these chapters, care has been taken to describe in sufficient detail what the systems are, as well as what their rheological properties are. Much attention is given to the variation of viscosity or stiffness with temperature, pressure, pH, degree of polymerization, breadth of polymer molecular-weight distribution curve and other aspects of physico-chemical constitution, and comparatively little space is devoted to the question of a more complete rheological investigation of any single sample.

The reader cannot but be impressed by the long lists of references to original papers throughout the volume, a high proportion of them in the literature of the last decade. Evidently, considerable selection of material has been necessary in the preparation of each chapter. The book should provide a useful introduction to the rapidly growing literature in branches of rheology which the reader is not actively engaged in, and will be a valuable work of reference for all physicists, physical chemists and applied mathematicians interested in deformation and flow problems.

The somewhat divergent lines of thought about rheological phenomena which are now current are brought together in this book, though in independent surveys with little attempt at co-ordination. The complete work might well help indirectly to bring some unity into the subject, but the work of integrating ideas put forward by different writers remains to be done.

J. G. Oldroyd